ISSN [Online]: 2583-2654

Observer reference Quantum tunneling through a dimensional square potential barrier under change in framework

Takele Teshome Somano
Department of Physics, College of Natural and Computational Sciences,
Wolaita Sodo University, Sodo, Ethiopia

Abstract: This study reports on quantum tunneling using a one dimensional (1D) square (SPB) curve under the Viewers Reference Frame (OFR). To this day, tunneling with SPB is considered with the notion that the OFR is the same for all tunnel measurements; therefore, the possibility of tunneling changes perceived as OFR variable has not been answered. In this paper, 1D SPBs are considered under OFR variability. Two types of OFR (periodic-square-wave variation and periodic-seen tooth-wave fluctuations) can be constructed by representing the particle transfer time via SBP. Under these types of variables, the intermediate transfer possibilities gradually increase the particle strength, which is associated with the transfer potentials to the permanent OFR state with a force much larger than the magnitude of the variable. In the case of periodic-square-wave waveform fluctuations the medium transmission potential is very high in the amplitude of the variables. Therefore, the ability of the medium to transfer cell energy is capable of reflecting the OFR flexibility distribution.

Keywords: Quantum Tunnelling, Potential Barrier, Viewer Effect, change of a viewer's Frame of Reference, viewing Frame of Reference

I INTRODUCTION

Tunnelling is the quantum mechanical phenomenon in which a particle passes through a potential barrier. It is fundamental to understanding the wave nature of particles [1-3] and plays a central role in various practical applications, such as in radioactive disintegration [4-11], electron tunnelling devices [12-15], quantum computation [16-22], and scanning tunnelling microscopy [23-29]. Particle tunnelling can occur in a potential barrier with a thickness in the order of nanometres or less [23-25]. Tunnelling can be estimated in terms of Heisenberg's uncertainty principle [3], which holds that measuring the momentum (position) of a particle can be conducted with precision, whereas the resulting position (momentum) disturbance of the particle is limited by Planck's constant divided by the uncertainty of the momentum (position) of the particle. When a particle is located at a potential barrier, it can pass through the barrier. This is because a particle has uncertainty in terms of its momentum; additionally, according to Heisenberg's uncertainty principle, the resulting position disturbance of the particle can be greater than the width of the barrier. For an example, a square potential barrier (SPB) provides an exact solution for calculating the transmission probability of a particle; it can quantitatively calculate tunnelling as well as provide a realistic approximation of particle tunnelling. In this paper, particle tunnelling was quantitatively studied according to the transmission probability of a particle through a potential barrier. To date, tunnelling has been examined under the assumption that an observer's frame of reference (OFR) remains constant. In much of the research, the OFR is assumed to be zero throughout the tunnelling measurements. Therefore, the change of the tunnelling probability when the OFR is assumed to

ISSN [Online]: 2583-2654

fluctuate currently remains unanswered. Recently, a novel observer effect induced by OFR a fluctuation was proposed in an Einstein solid [30] and a single-electron transistor (SET) [31]. The molar specific heat at a constant volume of an Einstein solid as a function of temperature revealed the distribution of OFR fluctuations,

which exhibited a peak and convergence of three times the level of the gas constant at low temperatures under periodic square-wave and periodic-saw tooth-wave fluctuations, respectively. Regarding the SET, the average current in an SET can also reveal the distribution of OFR fluctuations. An SET comprised a source, drain, and single channel. The average current in it exhibited an asymmetric zero-bias Coulomb peak as a function of the energy of the channel under periodic-square-wave and periodic-saw tooth-wave fluctuations—the amplitude of which gradually increased as the amplitude of the fluctuations increased. The amplitude of the zero-bias Coulomb peak was greater in the case of periodic-square-wave fluctuations. An observer effect induced by fluctuations of the OFR can be investigated if the reference of energy of a particle is matched to an OFR. In this study, the average transmission probability of a particle to transmit an SPB [32] was investigated under OFR fluctuations. The average transmission probability was formulated for two types of fluctuations in time representations, namely, periodic-square wave fluctuation and periodic-saw tooth-wave fluctuation. Under these types of fluctuations, the average transmission probability monotonically increases with the energy of the particle, which is saturated to the transmission probability in the case of a stationary OFR at a much greater energy than the amplitude of the fluctuations. The average transmission probability rapidly increases just above the energy corresponding to the amplitude of fluctuations of the OFR in the case of periodic square-wave fluctuations. Therefore, the average transmission probability with a particle's energy may be able to reveal the distribution of OFR fluctuations

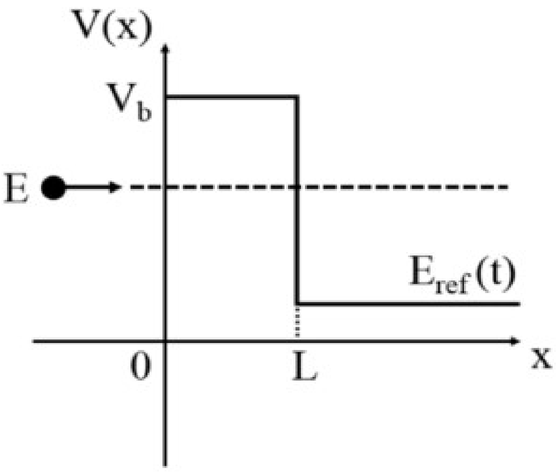


Figure1. Schematic of a 1D SPB, where V_b and L are potential energy and width of the barrier, respectively the energy of a particle $E_{ref}(t)$ is the energy of an OFR at time t. The transmission probability through the barrier was monitored by the observer on the right side of the barrier.

II AVERAGE TRANSMISSION PROBABILITY THROUGH A ONE-DIMENSIONAL SQUARE POTENTIAL BARRIER

Figure 1 shows a schematic of a one-dimensional (1D) SPB, where V_b and L are the potential energy and width of the barrier, respectively, E the energy of a particle, and $E_{ref}(t)$ the energy of an OFR at time t. In this study, $E_{ref}(t)$ was assumed to be constant at a time interval between and $t + \Delta t$, where d ≪ Δt ($= t_f - t_i$). Here, t_i and t_f are the initial and final times, respectively. If a particle from left (at x ≪ 0) was located at the 1D SPB, then the particle's transmission through the barrier was monitored by the observer far from the 1D SPB (at x ≫L).

ISSN [Online]: 2583-2654

In terms of the energy of a particle, $E \gg V_b$, the wave function of a particle under the 1D SPB at t,

$\psi(x,t) = \varphi(x)e^{-iEt/\hbar}$, where $\varphi(x)$ is expressed as

$$\varphi(x)\begin{cases} e^{ikx} + Re^{-ikx}\ for\ -\infty < x < 0 \\ Ae^{-x\alpha} + Be^{\alpha x} + for\ 0 \le x \le L \\ Te^{iqx} for\ L \le x < \infty \end{cases} \qquad (1)$$

Where $k = \sqrt{2mE/\hbar}$, $\alpha = \sqrt{2m(E - V_b)/\hbar}$, $q = \sqrt{2mE(t)/\hbar}$

Here, m is the mass of the particle, $\hbar$ the plank's constant divided by 2π, $E(t) = E - E_{ref}(t)$ the measured energy for E, at time t, and $\ge E_{ref}(t)$. R and T are the coefficients of reflection and transmission of the particle, respectively. The transmission probability of the particle through the 1D SPB, 1- $|R|^2$, can expressed as

(Please refer (2) equation in the last page of this article)

In terms of the energy of a particle, $0 < E < V_b$, the wave function of a particle under the 1D SPB at t, $\psi(x,t) = \varphi(x)e^{-iEt/\hbar}$, where $\varphi(x)$ is expressed a

$$\varphi(x)\begin{cases} e^{ikx} + Re^{-ikx}\ for\ -\infty < x < 0 \\ Ae^{-\beta x} + Be^{\beta x} + for\ 0 \le x \le L \\ Te^{iqx} for\ L \le x < \infty \end{cases} \qquad (3)$$

Where $\beta = \sqrt{2m(E - V_b)/\hbar}$. The transmission probability of the particle through the 1D SPB, can expressed as

(Please refer (4) equation in the last page of this article)

For the energy of an OFR at time t, $E_{ref}(t) \le E < V_b$, the average transmission probability through the 1D SPB with time interval Δt, 1- $|R_{avg}|^2$, can be expressed as

(Please refer (5) equation in the last page of this article)

For an observer with a stationary frame of reference, $(E_{ref}(t) = 0)$.the corresponding transmission probability, 1- $|R_0|^2$,can be expressed as

$$1\text{-}|R_0|^2 = \frac{4E(V_b - E)}{4E(V_b - E) + v_b^2 \sinh^2(L\sqrt{2m(V_b - E)/\hbar}} \qquad (6)$$

It has been found that the transmission probability of a particle in stationary OFR, 1- $|R_0|^2$, is 0 E=0 and then gradually increases with increasing E. In the limit of $\beta L \ll 1$, 1- $|R_0|^2$ 1(i.e., the barrier is sufficiently thin to be transparent). In the limit of $\beta L \gg 1$,1-$|R_0|^2 \approx \{16E(V_b - E)/v_b^2 \exp(-2L\sqrt{2m(V_b - E)/\hbar})$ (i.e., overall 1- $|R_0|^2$, exponentially increases with increasing E).

(Please refer Figure 2. (a) in the last page of this article)

Figure 2. (a) Fluctuating frame of reference at time t, $(E_{ref}(t)$, for periodic-square-wave fluctuations, of which the amplitude and period are $\varepsilon_1$and τ_1, respectively. Measured energy for E, at time t, E (t) (=E-$E_{ref}(t)$) and stationary frame of reference, E_{ref} which was considered to be equal to 0. (b) Fluctuating frame of reference at time t, $E_{ref}(t)$, for periodic-saw tooth-wave fluctuations, of which the amplitude and period are` $\varepsilon_2$and τ_2, respectively.

III AVERAGE TRANSMISSION PROBABILITY THROUGH A 1D SPB UNDER PERIODIC SQUARE-WAVE FLUCTUATIONS

ISSN [Online]: 2583-2654

Given periodic $E_{ref}(t) = \begin{cases} -\varepsilon_1 \, for\ 0 \le t < \tau_1/2 \\ \varepsilon_1 \, for\ \tau_1/2 \le t < \tau_1 \end{cases}$ (figure 2(a)), the measured energy for E, at time t, can be expressed as

$$E(t)=\begin{cases} E + \varepsilon_1 \, for\ 0 \le t < \tau_1/2 \\ E - \varepsilon_1 \, for\ \tau_1/2 \le t < \tau_1 \end{cases} \quad (7)$$

Where $\varepsilon_1$and τ_1 are the amplitude and period of the periodic-square-wave fluctuations, respectively.

3.1. Periodic-Square-Wave Fluctuations of a Half Period

For an observer with a fluctuating frame of reference by means of half-period periodic-square-wave fluctuations, the corresponding transmission probability, 1- $|R_{i,j}|^2$, is expressed as

(Please refer (8) equation in the last page of this article)

Where $t_i = 0\ and\ t_f = \tau_1/2\ .Here, j = 1.$

(Please refer (9) equation in the last page of this article)

Where $t_i = \tau_1/2\ and\ t_f = \tau_1 .Here, j = 2.$

3.2. Periodic-Square-Wave Fluctuations of One Period

For an observer with a fluctuating frame of reference by means of one-period periodic-square-wave fluctuations, the corresponding transmission probability, 1- $|R_1|^2$, is expressed as

(Please refer (10) equation in the last page of this article)

Where $t_i = 0\ and\ t_f = \tau_1$.

As shown in figure 3, the average transmission probability of a particle with an electron's mass, 1- $|R_1|^2$, is displayed as a function of E, where V_b= 1 eV [33]. 1- $|R_1|^2$ is 0 at E=0 and then gradually increases with to just above E= ε_1 and is saturated to the transmission probability in the case of the stationary OFR (`$\varepsilon_1 = 0$), 1- $|R_0|^2$. In the limit of $\beta L \ll 1$ and $\varepsilon_1 \ll< V_b$, 1- $|R_1|^2 \approx$ 1. As an example, for a SPB with L= 1 pm, the barrier is transparent at high energies, as shown in figure 3(a).In the limit of $\beta L \gg 1$and $E \ll \varepsilon_1$,

(Please refer equation 1- $|R_1|^2$ in the last page of this article)

For a SPB with L= 1 nm, overall 1- $|R_1|^2$ exponentially increases with increasing and is distinguished from 1- $|R_0|^2$, as shown in figure 3(b).

(Please refer Figure 4 in the last page of this article)

Figure 4. Average transmission probability of a particle with an electron's mass through the 1D SPB under the periodic-saw tooth-wave fluctuations, 1- $|R_2|^2$, as a function of E at (a) L =1 pm and (b) L =1 nm. Here, `ε_1=0.01 meV (light gray), 0.1 meV (gray), and 1 meV (black). The dotted lines denote 1- $|R_0|^2$ V_b is set as 1 eV.

IV CONCLUSION

In this paper, Quantum tuning with SPB was studied under the occasional variation of OFR. The probability of transmitting the average particle size is calculated by volume. In addition, the rate of transfer potential created for two types of time variables in the representation of time, namely, periodic-square-wave

ISSN [Online]: 2583-2654

fluctuations and periodic-saw dental fluctuations, based on the assumption that the particle index was compared to OFR. . Under the occasional square fluctuations - single-wave waves or high frequency limits, the median transmission potential was 0 at the particle strength, E = 0; increased gradually with the increase in particle strength, increased rapidly just beyond the magnitude of the variable, and filled with opportunities for transmission in the form of standing OFR. Similarly, under the variation of a periodically observed dental wave or at high frequency limits, the chances of medium transmission gradually increase with the particle strength but show a much smoother increase over the magnitude of the variability. Therefore, the possibilities for medium transfer with a strong particle SPB provide clear conditions for indicating the distribution of OFR variability.

Acknowledgements

this work was supported by my family.

REFERENCES

[1] L. de Broglie, "Recherches sur la théorie des quanta," Ann. Phys., vol. 10, 1925, pp. 22–128.

[2] E. Schrödinger, "An undulatory theory of the mechanics of atoms and molecules," Phys. Rev., vol. 28, 1926, pp. 1049– 1070.

[3] W. Heisenberg, "Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik," Z. Phys., vol. 43, 1927, pp. 172–198.

[4] J. Elster and H. Geitel, "Bequerel rays," Wied. Ann., vol. 66, 1889, p. 735.

[5] E. R. von Schweidler, Premier Congres de Radiologie, Liege, 1905.

[6] E. Rutherford, "Radioactive substance emitted from Thorium compounds," Phil. Mag., vol. 49, 1900, pp. 1–14.

[7] G. Gamow, "The quantum theory of atomic nucleus," Z. Phys., vol. 51, 1928, pp. 204–212.

[8] R. W. Gurney and E. U. Condon, "Wave mechanics and radioactive disintegration," Nature, vol. 122, 1928, p. 439.

[9] G. Gamow, "The quantum theory of nuclear disintegration," Nature, vol. 122, 1928, pp. 805–806.

[10] R. W. Gurney and E. U. Condon, "Quantum mechanics and radioactive disintegration," Phys. Rev., vol. 33, 1929, pp. 127– 140.

[11] C. Détraz, "The discovery of radioactivity: A one-hundred year heritage," Nucl. Phys. A, vol. 654, 1999, pp. 12c–18c.

[12] R. H. Fowler and L. Nordheim, "Electric emission in intense electric fields," Proc. Roy. Soc. A, vol. 119, 1928, pp. 173–181.

[13] L. Esaki, "Long journey into tunneling," Proc. of the IEEE, vol.62, 1974, pp. 825–831.

[14] I. Giaever, "Electron tunneling and superconductivity," Science, vol. 183, 1974, pp. 1253–1258.

[15] B. D. Josephson, "The discovery of tunneling super current, "Science, vol. 184, 1974, pp. 527–530.

[16] P. Benioff, "The computer as a physical system: A microscopic quantum mechanical Hamiltonian model of computers as represented by Turing machines," Journal of Statistical Physics, vol. 22, 1980, pp. 563–591.

[17] Y. I. Manin, "Computable and noncomputable (in Russian),"Sov. Radio. 1980, pp. 13–15.

[18] R. Feynman, "Simulating Physics with Computers, "International Journal of Theoretical Physics, vol. 21, 1982, pp.467–488.

[19] P. W. Shor, "Algorithms for quantum computation: discrete logarithms and factoring," Proceedings 35th Annual Symposium on Foundations of Computer Science, IEEE Comput. Soc., 1994, pp. 124–134.

[20] D. Loss and D. P. DiVincenzo, "Quantum computation with quantum dots," Phys. Rev. A, vol. 57, 1998, pp. 120–126.

[21] J. Clarke and F. K. Wilhelm, "Superconducting quantum bits, "Nature, vol. 453, 2008, pp. 1031–1042.

[22] E. Gibney, "Quantum gold rush: the private funding pouring into quantum start-ups," Nature, vol. 574, 2019, pp. 22–24.

[23] G Binnig, H. Rohrer, Ch. Gerber, and E. Weibel, "Tunnelling through a controllable vacuum gap," Appl. Phys. Lett., vol. 40, 1982, pp. 178–179.

[24] G. Binnig, H. Rohrer, Ch. Gerber, and E. Weibel, "Surface studies by scanning tunnelling microscopy," Phys. Rev. Lett., vol. 49, 1982, pp. 57–60.

[25] G. Binnig, H. Rohrer, Ch. Gerber, and E. Weibel, "7 × 7 Reconstruction on Si (111) resolved in real space," Phys. Rev. Lett., vol. 50, 1982, pp. 120–122.

[26] J. Tersoff and D. R. Hamann, "Theory of the scanning tunnelling microscope," Phys. Rev. B, vol. 31, 1985, pp. 805–813.

[27] M. Morgenstern, D. Haude, V. Gudmundsson, Chr. Wittneven, R. Dombrowski, Chr. Steinebach, and R. Wiesendanger, "Low temperature scanning tunnelling spectroscopy on InAs (110),"Journal of Electron Spectroscopy and Related Phenomena, vol.109, 2000, pp. 127–145.

[28] L. J. Lauhon and W. Ho, "Direct observation of the quantum tunnelling of single hydrogen atoms with a scanning tunnelling microscope," Phys. Rev. Lett., vol. 85, 2000, pp. 4566–4569.

ISSN [Online]: 2583-2654

(2) EQUATION

$$1-\ |R|^2 = \frac{8\alpha^2 kq}{(\alpha^2+k^2)+(\alpha^2+q^2)+4\alpha^2 kq-(\alpha^2-k^2)(\alpha^2-q^2)\cos(2\alpha L)}, \qquad (2)$$

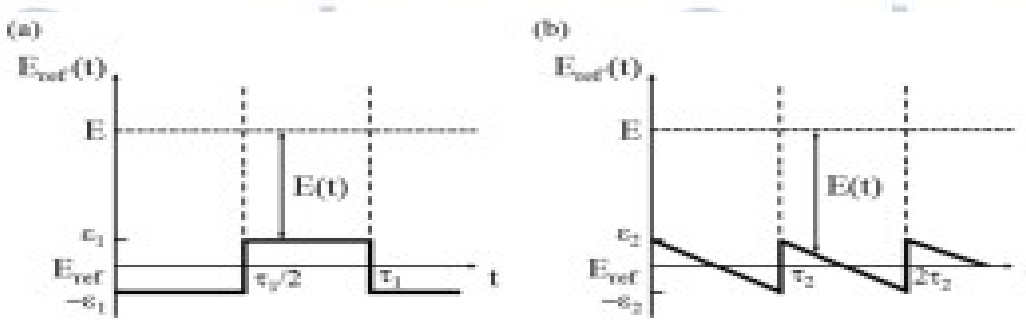
(4) EQUATION

$$1-|R|^2 = \frac{8\beta^2 kq}{(\beta^2+k^2)+(\beta^2+q^2)+4\beta^2 kq-(\beta^2-k^2)(\beta^2-q^2)\cos(2\beta L)} \qquad (4)$$

(5) EQUATION

$$1-\ |R_{avg}|^2 = \frac{1}{\Delta t}\int_{t_i}^{t_f} \frac{8\beta^2 kq}{(\beta^2+k^2)+(\beta^2+q^2)+4\beta^2 kq-(\beta^2-k^2)(\beta^2-q^2)\cos(2\beta L)}\,dt \qquad (5)$$

Figure 2 (a)



(8) EQUATION

$$1-\ |R_{1,1}|^2 = \frac{8(V_b-E)\sqrt{E(E+\varepsilon_1)}}{-(V_b-2E)(V_b-2E-\varepsilon_1)+4(V_b-E)\sqrt{E(E+\varepsilon_1)+V_b(V_b+\varepsilon_1)\cosh\left(2L\sqrt{2m(V_b-E)}/\hbar\right)}}, \qquad (8)$$

(9) EQUATION

$$1-\ |R_{1,2}|^2 = \frac{8(V_b-E)\sqrt{E(E-\varepsilon_1)}}{-(V_b-2E)(V_b-2E+\varepsilon_1)+4(V_b-E)\sqrt{E(E-\varepsilon_1)+V_b(V_b-\varepsilon_1)\cosh\left(2L\sqrt{2m(V_b-E)}/\hbar\right)}} \qquad (9)$$

ISSN [Online]: 2583-2654

(10) EQUATION

$$1-|R_1|^2 = \begin{cases} 0.5\left(1-|R_{1,1}|^2\right) + 0.5\left(1-|R_{1,2}|^2\right) \; for\; E \geq \varepsilon_1 \\ 0.5\left(1-|R_{1,1}|^2\right) \; for\; 0 \leq E < \varepsilon_1 \end{cases}, \quad (10)$$

(1-$|R_1|^2$) EQUATION

$$1-|R_1|^2 \approx \left\{8(V_b - E)\sqrt{E\varepsilon_1/v_b^2}\right\} exp\left(-2L\sqrt{2m(V_b - E)/\hbar}\right).$$

Figure 4